

# Galactic disks as reaction-diffusion systems

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## ABSTRACT

A model of a galactic disk is presented which extends the homogeneous one zone models by incorporating propagation of material and energy in the disk. For reasonable values of the parameters the homogeneous steady state is unstable to the development of inhomogeneities, leading to the development of spatial and temporal structure. At the linearized level a prediction for the length and time scales of the patterns is found. These instabilities arise for the same reason that pattern formation is seen in non-equilibrium chemical and biological systems, which is that the positive and negative feedback effects which govern the rates of the critical processes act over different distance scales, as in Turing's reaction-diffusion models. This shows that patterns would form in the disk even in the absence of gravitational effects, density waves, rotation, shear and external perturbations. These nonlinear effects may thus explain the spiral structure seen in the star forming regions of isolated flocculent galaxies.

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# 1 Introduction

Problems of pattern formation occur on many scales in astronomy, from the large scale distribution of galaxies to the formation of stars and planets. Perhaps the most studied of these is the problem of the formation of spiral structures in galactic disks. This has been approached by a variety of theoretical and numerical tools [1, 2, 3, 4, 5, 6, 7]. However, despite the partial success of some of these models, there seem to be important features of the phenomena that are so far unexplained. These have to do mainly with the ubiquity, stability and persistence of the structures seen in the star forming regions of spiral galaxies. The density wave models[2], while successful in some cases, have difficulty explaining why structure should persist in isolated galaxies for times much longer than a few rotations of the disk[7]. On the other hand, models based on the notion of propagating star formation[1], while successful in reproducing the appearance of a range of galaxies[3, 4], involve either drastic simplifications of the astronomy or require fine tuning of rates in order to be able to reproduce the observed structures. Because of these limitations, it may not be inappropriate to consider new approaches to this problem.

Before beginning, it is important to emphasize that the problem of spiral structure in galaxies is in fact several distinct problems. There is a problem of transitory structure, which is caused by excitations of modes of the disk, most likely by encounters with other galaxies. (This is called *galactic harassment* in [8].) There is both observational and theoretical evidence that grand design spirals are the result of such phenomena[7, 8]. Such transitory structure are most likely correctly seen to involve density waves, and are outside of the phenomena to be considered in this paper. On the other hand, there is a class of spiral galaxies in which the spiral structure is primarily an aspect of the star formation process. A typical galaxy in this class is an *Sc* galaxy with flocculent spiral structure, in the field, sufficiently far from other galaxies that the observed spiral structure must be endogenous, which is to say it must be understood as a product of processes occurring in the disk. Such galaxies typically show spiral structure in blue light, but not in red light[6, 3]. This indicates that the observed structure is primarily not a density wave in the disk, but is instead a trace of the star formation process.

In such galaxies the star formation process proceeds, as far as is known, at a constant rate when averaged over the whole disk[6]. As a result, the ratio of the present star forming rate to the rate averaged over the history

of the galaxy is one. This constancy of the star formation rate is by itself a significant clue, as it implies that there must be feedback mechanisms in the processes that govern the rate of star formation, so as to keep the rate slow and constant[9, 10]. Given the fact that the rates are constant over time scales much longer than those of the dynamical processes involved ( $10^{10}$  years versus at most  $10^7$  years), there is no other way one could explain the fact that galaxies with slow and steady rates of star formation are so common.

Other evidence concerning this will be discussed below, but the overall conclusion will be that the star formation process can be understood to be one component a network of self-regulated and autocatalyzed processes which is the result of the self-organization of the material in the galactic disk. Thus, the disk may be understood as a far from equilibrium statistical system in which a network of processes involving flows of energy and materials among its several components has arisen which is governed by a set of feedback loops. Once this is seen, it is clear that the patterns produced by the star formation process might be understood in the context of the way in which spatial inhomogeneities are produced and stabilized in the context of such non-equilibrium systems.

In recent years a body of theoretical and experimental work has grown up which studies the problem of how spatial and temporal patterns are produced in non-equilibrium systems [12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. The systems of interest include many inorganic systems such as the BZ reaction[19], diffusion limited aggregation[20] and self-organized critical systems[21]. It includes as well numerous biological systems such as colonies of bacteria[18], the differentiation of cell types[14] and the formation of structure in the embryology of multicellular creatures[13, 14]. Recently the study of such systems has produced many successes by which patterns that can be reproduced at will in the laboratory are explained by simple models. These models typically involve both partial differential equations and discrete elements such as cellular automata. As a result, one can begin to speak of the existence of a paradigm of structure formation in far from equilibrium steady state systems.

Most of this work, and certainly the bulk of the impressive results, post-dates the formation of the main ideas that have governed thinking about structure formation in astronomy. As such it is reasonable to assess whether anything has been learned there that could be of use in understanding problems of structure formation of astrophysics. While it is premature to answer generally, the purpose of this paper is to suggest that the answer is yes for

the particular case mentioned above, of an *Sc* galactic disk, far from other galaxies, with flocculent spiral structure.

To see why this might be the case, we may list the main elements that characterize systems to which non-equilibrium models of pattern formation apply. These include[12]-[21]

- The system is in a steady state, with a slow (relative to the relevant dynamical time scales) and steady flow of energy, and perhaps matter running through it.
- The steady state is far from thermodynamic equilibrium. There is a coexistence of several species or phases of matter, which exchange matter and energy among themselves through closed cycles.
- The rates at which material flows around these cycles is governed by feedback loops that have arisen during the organization of the system to the steady state.
- The reaction networks of these cycles are *autocatalytic*. This means that any substances that serve as catalysts or repressors of reactions in the network are themselves produced by reactions in the network.
- There may be spatial segregation of the different phases or materials in the cycles. This occurs when the inhibitory and catalytic influences propagate over different distance scales. At the smallest scale this means that the production of certain substances may be subject to refractory periods, so that once production has occurred in a local region, it will not be repeated there for a certain period of time.

Given a system with these characteristics, there are models available which describe how spatial structure is formed and stabilized. These come in several varieties, but the most typical are called reaction-diffusion systems[12, 13, 14]. They have been used to explain the occurrence of spatial structure in inorganic systems such as the *BZ* reaction[19]<sup>1</sup> as well as in organic systems such as patterns on sea shells, the stripes and spots on the coats of mammals and the segregation of cell types in embryology[12, 13, 14].

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<sup>1</sup>Note that I am not arguing that there is an analogy between this and galactic disks because in both cases the patterns formed are spirals. This is a superficial resemblance, which is due mostly to the fact that the disk is rotating differentially. The useful analogies are the ones I have described here.

The next section begins by reviewing evidence that spiral galaxies of the kind we are considering fit these criteria point for point. This leads to a description of the galactic disk as an autocatalytic network of reactions. A first step towards a model of the reaction network is made, which resembles a homogeneous “one zone model”. The model is analyzed to show that the system reaches a steady state in which condensation of GMC’s and star formation proceed at rates determined by a balance of positive and negative feedback. In section 3 and 4 this model is extended to allow spatial variation, and it is observed that there is an alternating pattern of negative and positive feedback as one proceeds from the largest to the smallest scales. Thus, the technology of reaction diffusion models[12, 13, 14] is appropriate to the study of structure formation. The linearized analysis of this model is carried out in section 5. The conclusion is that for astrophysically reasonable values of the parameters the homogeneous state is unstable, leading to the initiation of pattern formation. Directions for future work are then discussed in the conclusion.

## 2 Galactic disks as autocatalytic systems

When a non-equilibrium chemical system organizes itself there arise cycles of reactions by which the energy and material inputs to the system are transformed into various aggregates in a steady state. It is, indeed, not difficult to show that non-equilibrium steady state systems will generically evolve to the point where such cycles develop. An important feature of chemical reactions is also that many reactions are catalyzed or inhibited by other elements. When these catalysts or inhibitors are produced in the same network of chemical reactions in which they act we say that the network is autocatalytic. Autocatalytic networks are ubiquitous features of biological and non-equilibrium chemical systems. They are particularly important because feedback occurs naturally in autocatalytic networks.

A first step to applying the pattern formation paradigm to galactic disks is to see that the interstellar medium is the site of a number of processes, which form a network analogous to a network of chemical reactions. Further, all of the significant processes involved in star formation are either catalyzed or inhibited by the products of processes in the disk, so that a galactic disk may usefully be analyzed as an example of an autocatalytic reaction network.

Progressing from larger to smaller scales, we may list some of the main reactions involved in the star formation process, together with their catalysts

or inhibitors.

## 2.1 Condensation of giant molecular clouds (GMC's)

These cold clouds condense out of the ambient interstellar medium, (ISM) forming apparently scale invariant, or fractal distributions of very cold molecular gas and dust.

- **Catalysts:** The main catalysts involved are dust, as well as carbon and oxygen. Dust is produced mainly in the atmospheres of cool giant stars while carbon and oxygen are produced by fusion in stars. The dust shields the clouds, allowing them to cool even in the presence of  $uv$  radiation from massive stars. The dust grains also serve as sites for molecular binding. The carbon and oxygen are apparently necessary to cool the clouds, as radiation from rotational modes of  $CO$  is apparently the main cooling mechanism.

We may note that these substances spread through the *ISM* over intermediate distance scale  $L_{int}$ , corresponding to the scale over which the products of supernovas and massive stars are spread through the interstellar medium by shock waves from supernovas. A rough scale for  $L_{int}$  is 100 parsecs.

- **Inhibitors:** The main inhibitor to the process is ultraviolet radiation from massive stars. This heats the ambient ISM, making condensation less probable. The picture of this process proposed by Parravano and collaborators is very useful [10]. They argue that there is a phase boundary separating an ambient phase, in which the medium consists of warm atomic gas and a condensed phase, in which it consists of cold molecular gas. The phase boundary is a curve in the  $P-T$  plane, which we may denote  $T_c(P)$ . There is then a simple feedback process which keeps the medium on the phase boundary, at which *GMC's* condense at a steady rate. The pressure of the medium is determined by the supernova rate, as the ionized regions formed by the supernovas are the main source of pressurization. Thus, the average pressure is a constant in the steady state, and is determined by the supernova rate. On the other hand the temperature is due to a competition between radiation from the disk and heating, the main source of which is the  $uv$  radiation from massive stars.

Given this information it is clear the system will evolve to the phase boundary. If the massive stars heat the medium to a temperature greater than  $T_c(P)$  no more clouds condense. Then after a time  $\tau$  on the order of  $10^7$  years, a typical lifetime of a massive star, the medium will begin to cool as a result of a decline in the  $uv$  radiation as the massive stars that are its source supernova<sup>2</sup>. But when the temperature falls below  $T_c(P)$  then more clouds condense, leading to the formation of more stars and hence more  $uv$  radiation.

The length scale for this inhibition process is  $L_{long}$ , which is much larger than  $L_{int}$  and may be as large as the radius of the stellar disk. The time scale is relatively short, some  $10^4$  years, which is the time it takes the radiation to spread through the disk.

We may note that the hypothesis that the condensation of the GMC's takes place mainly on the critical curve may account for the fact that the density distribution is scale invariant. This process may then be seen as an example of self-organized criticality, by which a non-equilibrium system evolves to a steady state in which the spatial and temporal structure is described by power law correlations[21].

## 2.2 The collapse of GMC cores

The process of star formation begins when the cores of GMC's collapse. There may be some small spontaneous rate for core collapse, but the collapse of cores massive enough to lead to the formation of massive stars is apparently usually catalyzed.

- **Catalysts:** The main catalyst for core collapse is a shock wave coming from either a supernova or HII regions. Both of these are products of massive stars. These processes take place over the intermediate scale,  $L_{int}$ .
- There are also processes by which the impact of a density wave on a GMC may cause core collapse. In grand design galaxies these are thought to be important, but they are of lessor importance for flocculent galaxies of the type in which we are interested.

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<sup>2</sup>Of course, the supernovas heat the medium temporarily in a local region, but over larger scales the heating is mainly through the  $uv$  radiation from the massive stars. In any case if there is no new formation of massive stars the medium must cool, which will be after a time on the order of  $10^7$  years.

- **Inhibitors:** The main inhibition to core collapse comes from two sources. Stellar winds from young massive objects disrupt the molecular clouds in which they formed. There is also evaporation of the *GMC*'s due to the *uv* light from young massive clouds formed. These processes both take place over a relatively short scale  $L_{short}$  not larger than the size of one cloud complex. While related, this is a different process from that by which the *uv* light heats the ambient medium. We may note that these processes are effective enough that the star forming efficiency of a given cloud is low, of the order of a few percent. The effect of this process has been described in terms of a latency time  $\tau_L$ . For any given region of the disk, the star forming process will not generally recur for a time greater than  $\tau_L$ , which is the time it takes for GMC's to begin to condense out of the gas, after a first GMC has been evaporated as a result of radiation produced by stars formed in it.

### 2.3 Star formation

The final stages of star formation involve the formation of a protostar and accretion disk. This process is self-limiting, according to [11] matter accretes onto the protostar until it is stopped by outflow from the young star or its accretion disk. The source of this outflow may be a stellar wind from the accretion disk, which has been heated after the ignition of nuclear reactions. These stellar winds also cause shock waves that may heat and disperse the molecular clouds, further contributing to the inhibition of star formation.

### 2.4 A model

We may note that all of the catalysts and inhibitors involved in the star formation process, with the sole exception of the role that density waves play in initiating core collapse, are themselves products of stars. These are produced by a number of processes by which stars return matter and energy to the interstellar medium. We have already mentioned the most important of them above. They include supernova, which produces shock waves, enriched material and dust, and massive stars, which produce *uv* light, dust and material via evaporation and shock waves via the formation of ionized regions. Thus, it is clear that the reactions that take place in the disk of a spiral galaxy can be called an autocatalytic reaction network by analogy to chemical reaction networks.



If we don't take into account spatial variation, the reaction network may be described by a "one-zone model" which is analogous to the systems of equations that describe homogeneous chemical reaction networks. Making reasonable assumptions we arrive at a system of equations, of the type familiar from one zone models. If we label the densities of the components as

$$\begin{aligned}
c &\equiv \text{cold gas in GMC's} \\
w &\equiv \text{warm ambient gas} \\
s &\equiv \text{massive stars} \\
d &\equiv \text{light stars} \\
r &\equiv \text{density of uv radiation}
\end{aligned}$$

We have,

$$\dot{c} = \frac{\alpha' w^2}{r} - \frac{\beta}{1 + \kappa s} cs - (\gamma + \mu) cs \quad (1)$$

$$\dot{s} = \frac{\beta}{1 + \kappa s} cs - \frac{s}{\tau} \quad (2)$$

$$\dot{w} = -\frac{\alpha' w^2}{r} + \frac{s}{\tau} + \gamma cs + \delta \quad (3)$$

$$\dot{r} = \eta' s - \phi' wr \quad (4)$$

$$\dot{d} = \mu cs \quad (5)$$

The model contains a number of parameters that describe the rates of the various astrophysical processes.  $\tau$ , which will turn out to govern the characteristic time scale of the instabilities, is approximately the lifetime of a typical massive star, which will be taken to be  $10^7$  years. (More precisely it gives the rate of flow of matter from the massive stars to the warm gas, whether that takes place during the supernova or earlier by evaporation.)  $\alpha'$  is proportional to the characteristic rate for GMC's to condense from the warm ambient gas. As the efficiency of the formation of massive stars, as well as the infall rate are small, we must choose parameters such that in the steady state,  $\alpha' w/r > \tau^{-1}$ .

Three of the parameters,  $\beta, \gamma$  and  $\mu$  govern the rates per unit mass density at which material flows from the GMC's to other of the states of the ISM by processes which are catalyzed by the action of massive stars.  $\beta$  and  $\mu$  are the rates for formation of massive and light stars, respectively, per unit density of cold gas.  $\gamma$  is the rate per unit density that the cold gas

is heated by the effects of massive stars. Finally  $\delta$  is the rate at which warm gas accretes onto the disk by infall from outside the galaxy.

There is an additional parameter  $\kappa$  which represents the fact that massive stars cannot so easily form near other massive stars because the gas in their neighborhood will be heated and ionized. We may note that there already is a feedback effect whereas massive stars induce a heating of cold gas to warm gas, and thus suppress locally their own formation. However, there are also additional effects such as the fact that shock waves from supernova and *HII* regions will evaporate *GMC*'s close to the star, while they catalyze collapse of *GMC* cores further, after they have slowed down. This effect is local on the scale of the model, and hence may be represented by the parameter term  $\beta/(1 + \kappa s)$  which represents a local negative feedback for the formation of massive stars.

The model is of course based on several simplifications. To begin with the continuous spectra of stellar masses is reduced to two types.  $s$  measures the density of matter in massive stars, by which is meant stars massive enough to supernova, while  $d$  measures the density of matter in light stars, which are stars too light to supernova. The fact that light stars do return matter to the ISM by evaporation is one of the processes that is ignored here. Another is that there may be also a process by which light stars condense spontaneously from the *GMC*'s. Still another is the fact that the role of dust and carbon is not explicitly included, if they were this would lead to metals dependent corrections to the coefficients.

In addition, the Parravano process, by which the action of the  $uv$  radiation is hypothesized to keep the ISM at the phase transition between warm and cold gas is replaced here by a simple negative feedback, by which the condensation of cold gas is suppressed by a factor of  $r^{-1}$ , which represents the density of  $uv$  radiation. The reason is that a system with this form of feedback evolves to a steady state at which the flows of material between the different phases are constant. A mechanism such as that proposed by Parravano, like any thermostat, evolves to a time dependent state which fluctuates around the equilibrium point. Thus, this kind of model is easier to use to study the hypothesis that instabilities in the steady state lead to the initiation of pattern formation.

A further simplification is that warm gas is assumed to fall onto the stellar disk from outside the galaxy homogeneously at a constant rate, whereas inflow from a larger gaseous disk is possibly at least as significant.

Even with these limitations, the system of equations (1-5) does give us an acceptable first model of how the ISM arrives at a steady state at which the

rates of its processes are determined by a balance of negative and positive feedback. Solving them for the steady state at which all time derivatives except  $\dot{d}$  vanish we find a unique solution given by

$$c_0 = \frac{1 + \kappa s_0}{\beta \tau} \quad (6)$$

$$s_0 = \frac{1}{2\kappa} \left( \sqrt{1 + \frac{4\delta\beta\tau\kappa}{\mu}} - 1 \right) \quad (7)$$

$$w_0^3 = \frac{\beta + \gamma}{\alpha' \beta \tau} s_0^2 + \frac{\delta}{\alpha'} s_0 \quad (8)$$

$$r_0 = \frac{\eta}{\phi'} \frac{s_0}{w_0} \quad (9)$$

We may note the  $\kappa$  independent relation

$$s_0 = \frac{\delta}{\mu c_0}. \quad (10)$$

We see also that

$$\dot{d} = \delta \quad (11)$$

so that in the steady state forms light stars at the inflow rate  $\delta$ . This is of course necessary, the light stars are simply a sink into which mass flows. At a steady state the mass that flows into the system must be equal to the mass that flows out of it, thus the rate of creation of light stars must equal the rate that matter accretes or flows onto the star forming regions of the disk.

### 3 The hierarchy of scales of inhibition and catalysis

System of equations such as (1-5) may be useful to understand why galactic disks organize themselves into steady states in which the average star formation rate is constant. As such they may be useful for such things as modeling the chemical evolution of the galaxy. Of course, none of this is new, as there is a large literature on one zone models and galactic evolution. What we may learn from the analogy to autocatalytic reaction networks in chemistry and biology is that there is a natural way to understand the generation of spatial and temporal patterns given such a reaction network. As stated in the introduction, what is needed is only that the substances which serve as

catalysts and inhibitors spread through the system over different distance and time scales. As shown in many examples [12, 13, 14, 17, 18, 19, 24], if there is a hierarchy of scales over which catalytic and inhibitory reactions are alternatively more important, one generally gets the formation of structure.

The main thesis of this paper is that this paradigm may be applied to arrive at a natural understanding of the occurrence of structure in galactic disks. To see that the preconditions are met, we may note that in the proceeding summary of the galactic reaction network we have had to distinguish three distance scales.  $L_{long}$  is the scale of the whole stellar disk;  $L_{int}$  is on the order of the distance between cloud complexes and  $L_{short}$  is the scale of a typical cloud. In fact, as the distribution of clouds may be scale invariant, these distance scales are characterized more exactly by the processes that give rise to them:  $L_{long}$ , the heating of the ambient medium by  $uv$  light;  $L_{int}$ , the distance over which a supernova may catalyze core collapse and  $L_{short}$ , the range over which a new star may evaporate the GMC out of which it condensed.

We may indeed observe that catalytic and inhibitory reactions alternate in importance. The processes that are characterized by  $L_{int}$  are all catalytic. This is the scale over which shock waves from supernovas and ionization regions induce new star formation in neighboring GMC's, and it is also the scale over which those processes distribute the dust and enriched material produced by stars and supernovae.

On the other hand, both the short and the long distance scales are dominated by inhibitory processes.  $L_{long}$  characterizes the heating of the ISM by  $uv$  light, which has the effect of suppressing the formation of new GMC's. The dominant effect on  $L_{short}$  is the inhibition of core collapse by the evaporation of a GMC by massive stars born there. It is also as a result of this that the latency period is associated with  $L_{short}$ .

## 4 Modeling the galactic disk as a reaction diffusion system

The conditions for structure formation in autocatalytic networks are clearly met. The issue is then how to model these processes and whether all the relevant physics has been incorporated so that the models reproduce the systematics of  $Sc$  flocculent spirals well.

The most common way to incorporate spatial inhomogeneities in pattern formation models is through diffusion. This is appropriate for biological

systems, and may play a role here, as there will clearly be some diffusion of stars, clouds and materials through the disk. However, this may not be sufficient for our purposes, as it is clear that effects such as shock waves or the influence of  $uv$  radiation are best treated in terms of propagation rather than diffusion.

It may however be that to a first approximation propagating star formation may be treated as a diffusion process. This is suggested by the phenomenological success of two different kinds of models, in which propagating star formation is modeled. These are the Gerola-Seiden-Schulman model based on a cellular automata[3] and the more sophisticated Elmegreen-Thomasson model[4]. In each of these the shock waves responsible for propagating star formation are not modeled directly, instead one models either a direct effect by which star-forming regions catalyze the initiation of star formation in neighboring regions (in the first case) or a process by which “young stars” diffuse from the clouds in which they are born and then give up their energy to nearby GMC’s (in the second).

The success of these models suggests that a diffusion-reaction model might be applicable to structure formation in spiral disks. To investigate this we may extend the homogeneous model described above by adding diffusion terms for the massive stars and the radiation field. To simplify the analysis that follows, we will reparameterize the model, by eliminating the parameters  $\beta, \gamma, \eta$  and  $\mu$  in favor of the homogeneous steady state values  $c_0, s_0, w_0$  and  $r_0$ . We then normalize all quantities so that

$$\bar{c}(x, t) \equiv \frac{c(x, t)}{c_0} \quad (12)$$

and likewise for the other quantities.

We then arrive at the system of equations,

$$\dot{\bar{c}} = \alpha \left[ \frac{\bar{w}^2}{\bar{r}} - \bar{c}\bar{s} \right] \quad (13)$$

$$\dot{\bar{s}} = D_s \nabla^2 \bar{s} + \frac{\bar{s}}{\tau} \left( \bar{c} \left( \frac{1 + \kappa s_0}{1 + \kappa s} \right) - 1 \right) \quad (14)$$

$$\dot{\bar{w}} = -\frac{\alpha c_0}{w_0} \left[ \frac{\bar{w}^2}{\bar{r}} - \bar{c}\bar{s} \right] + \frac{s_0}{w_0 \tau} \bar{s} [1 - \bar{c}] + \frac{\delta}{w_0} [1 - \bar{s}\bar{c}] \quad (15)$$

$$\dot{\bar{r}} = D_r \nabla^2 \bar{r} + \phi [\bar{s} - \bar{r}\bar{w}] \quad (16)$$

$$\dot{\bar{d}} = \delta \bar{c}\bar{s} \quad (17)$$

Here we have rescaled two parameters,

$$\alpha = \alpha' \frac{w_0^2}{c_0 r_0} \quad (18)$$

$$\phi = \phi' w_0 \quad (19)$$

Note that both now have dimensions of inverse time.  $\alpha$  gives the rate of condensation of cold clouds, while  $\phi$  gives the rate of energy loss of the radiation into the warm gas.

We have also introduced two diffusion constants,  $D_s$  and  $D_r$  which govern the diffusion of the effects of the massive stars and radiation, respectively. It is reasonable to take

$$D_s = \frac{L_{int}^2}{\tau} \quad (20)$$

The value of  $D_r$  may be taken to be much larger, perhaps on the order of the radius of the galaxy times the speed of light, corresponding to the fact that the  $uv$  radiation propagates through the whole disk.

As the disk is thin, it is sufficient to consider this as a model in two spatial dimensions.

We have thus arrived at a reaction diffusion model governing the dynamics of the interstellar medium. Of course, it leaves out many aspects of the physics of the galactic disk, such as gravitational effects that may lead to density waves and external perturbations. We have also so far not included the fact that the material in the galaxy is differentially rotating.

As the combinations of these effects is already known, under appropriate circumstances, to cause the temporary formation of spiral patterns, a complete model must take them into account. At the same time, we are interested in the regime discussed in the introduction, in which internal non- equilibrium processes in the disk are expected to be the cause of the observed patterns. This is the physics that we hope is captured in the model we have so far. Thus, our first task is to see if the ISM, as described by this model, will in fact develop structure. After we have done this we can advance to a more complete model in which rotation and gravitational effects are included.

Once we have the equations, the next step is to make a linearized analysis of the theory. This will allow us to discover if there are unstable modes which may develop into spatial structure.

## 5 Linearized analysis of instabilities

To proceed we now expand to linear order in perturbations from the steady state by writing

$$\bar{s} = 1 + S \quad (21)$$

and likewise for the other quantities. We arrive at a system of linear equations

$$\dot{C} = \alpha [2W - R - C - S] \quad (22)$$

$$\dot{S} = D_s \nabla^2 S + \frac{1}{\tau} C - S \frac{\nu}{\tau} \quad (23)$$

$$\dot{W} = -\frac{\alpha c_0}{w_0} [2W - R - C - S] - \frac{s_0}{w_0 \tau} \bar{s} C - \frac{\delta}{w_0} [S + C] \quad (24)$$

$$\dot{R} = D_r \nabla^2 R + \phi [S - R - W] \quad (25)$$

$$\dot{d} = \delta + C + S \quad (26)$$

where the strength of the local saturation effect is labeled by

$$\nu = \frac{\kappa s_0}{1 + \kappa s_0} \quad (27)$$

We may now look for solutions to the linearized equations of the form

$$\begin{aligned} C &= \mathcal{C} e^{\lambda t} \cos(k \cdot x) \\ S &= \mathcal{S} e^{\lambda t} \cos(k \cdot x) \\ W &= \mathcal{W} e^{\lambda t} \cos(k \cdot x) \\ R &= \mathcal{R} e^{\lambda t} \cos(k \cdot x) \end{aligned} \quad (28)$$

which would describe an instability with a wavevector  $\vec{k}$  growing exponentially with a time scale  $\lambda^{-1}$ . To find out if there are instabilities we must discover if there are such solutions for reasonable values of the parameters and reasonable wavelengths, in which  $\lambda$  is real and positive.

Using the ansatz (28) we have an eigenvalue problem,  $\mathcal{M}^a{}_b V^b = \lambda V^a$ , where  $V^a = (C, S, W, R)$  and the matrix is given by

$$\mathcal{M}^a{}_b = \begin{bmatrix} -\alpha & -\alpha & -2\alpha & -\alpha \\ 1/\tau & -k^2 D_s - \frac{\nu}{\tau} & 0 & 0 \\ \epsilon \alpha - \frac{\rho}{\tau} - \epsilon T^{-1} & \epsilon \alpha - T^{-1} & 2\epsilon \alpha & \epsilon \alpha \\ 0 & \phi' & -\phi' & -D_r k^2 - \phi' \end{bmatrix} \quad (29)$$

Here we have introduced new parameters.

$$\epsilon = \frac{c_0}{w_0} \quad (30)$$

is the ratio of cold gas to warm gas in the homogeneous solution, and should be about unity, or a bit less, as about half the gas in the ISM is observed to be in the GMC's.

$$\rho = \frac{s_0}{w_0} \quad (31)$$

is the ratio of mass in massive stars to mass in warm gas. It is small, perhaps about .1 reflecting the facts that the efficiency of the star formation process is low and the production of massive stars is suppressed by the power law in the initial mass function.

$$\frac{1}{T} = \frac{\delta}{w_0} \quad (32)$$

is the time scale for accretion of warm gas from outside the stellar disk. It is much longer than  $\tau$ , by a factor of  $10^2$  to  $10^3$ .

It is straightforward to study the behavior of the eigenvalues of this matrix as a function of the parameters. I describe the behavior for a typical set of parameters, and leave a more systematic discussion for further analysis. Some reasonable astrophysical values for the parameters are to take equal average densities for warm and cold gas ( $\epsilon = 1$ ), the rate of condensation of cold gas to be about ten times that of the return rate from massive stars to the warm gas ( $\tau\alpha = 10$ ),  $\rho$ , the ratio of matter in massive stars and warm gas at .1, and for the others,  $T = 100\tau$ ,  $D_r = 10^4 D_s$  and  $\nu = .5$ . There is one very negative eigenvalue, corresponding to the fast homogenization of the radiation field, which is a result of the large value of  $D_r$ . There are then three positive eigenvalues. This most positive eigenvalue governs the evolution of the dominant instability of the disk. In Figure 1 we graph the real part of the largest positive eigenvalue as a function of the wavelength  $l = 1/|k|L_{int}$ . We see that the most unstable modes are at short scales on the order of  $L_{int}$ . We don't trust the model for smaller scales, so what we can say from the graph is that with the parameters as chosen the model generates instabilities over all scales, with the fastest growing instabilities between  $L_{int}$  and about  $20L_{int}$ .

The time scale of the instability is about  $10^6$  years. Thus, within the limitations of the model, we may say that the scales of these instabilities are right to be the initiation of the formation of spiral patterns as seen in the flocculent galaxies. However to understand the formation and evolution



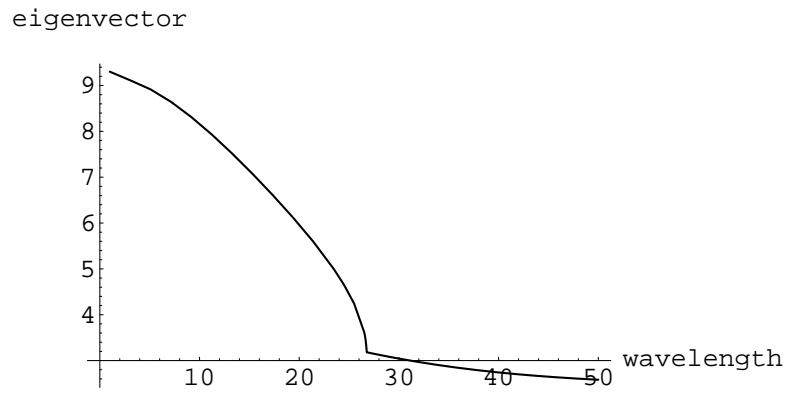


Figure 1: Dependence of the most unstable eigenvalue on wavelength, in units of  $L_{int}$ . The eigenvalue is in units of  $10^{-7}sec^{-1}$ .

of these patterns beyond this initial stage we need to carry out the full non-linear analysis.

## 6 Conclusions

The results of the linearized analysis support the basic theses put forward in the earlier sections. First, that a galactic disk can be seen as an autocatalytic network of reactions in which structure forms and persists because catalysis and inhibition (or positive and negative feedback) alternatively dominate as one descends from larger to smaller length scales. This means that patterns spontaneously form in the medium, even before we take into account gravitational effects, density waves, rotation or external perturbations. This suggests that the general framework of reaction- diffusion models may be appropriate for an understanding of the phenomenology of spiral galaxies. Further, the time and length scales of the instabilities are reasonable.

However, these results must be considered preliminary. Rotation must be added to the model, after which a full non-linear analysis must be carried out for a variety of parameters, and compared with structure observed in galaxies.

One flaw of the model which is visible even at the linearized level is that it does not allow a clear separation between local inhibitory effects and intermediate scale catalytic effects. Thus, near massive stars uv radiation and shock waves act to destroy GMC's, while further away at the intermediate scale the shock waves catalyze further star formation. As pointed out by Anderas Freund [22], this distinction is not represented in the model, which results in the fact that there is no suppression of the instability at short scales. This defect can be remedied by the explicit introduction of shock waves, so that the local effect of the star on its surroundings can be cleanly separated from the effect of its shock waves at intermediate scales. Preliminary results[22] show that when this is done there is a clear separation of small scale suppression from medium scale excitation resulting in a short distance cutoff for the scale of the instabilities.

Even with these improvements it may be expected that this kind of approach could successfully describe only the type of galaxies mentioned above, (flocculent Sb or Sc spirals, isolated from other galaxies) as these are the cases in which spiral structure is seen only in the star forming regions and not in the density of old stars. Still success in this domain would support the theses of this paper. While other models of these kinds of

galaxies exist which do successfully reproduce the observed structure[3, 4] these involve either drastic simplification of the physics or fine tuning of parameters. The model proposed here, if successful would explain how the system of the galactic disk organizes itself to tune its own parameters to a state characterized by a constant overall rate of star formation in which the star forming regions produce slowly evolving spiral patterns.

To gain a complete understanding of the dynamics of spiral disks, the model will have to be extended to include gravitational and density effects as well as rotation. The goal in the end will be an understanding of how effects of gravitation and rotation couple with the processes described here to produce the whole range of types of spiral structures. In particular, it might very well be the case that turbulence in the interstellar medium plays a role in star formation over a range of scales[23]. However this may not be as much a competing hypothesis as complementary to the ideas discussed here, given the fact that a transition has been observed in the BZ system[19] in which perfect spirals are disordered by defects leading to a kind of turbulent behavior reminiscent of the patterns seen in flocculent galaxies[24].

Finally, it may be that similar methods could lead to an understanding of other astrophysical problems involving pattern formation such as non-linear processes in galaxy formation.

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